

# Halo Star Evolution

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## Abstract:

The low mass, low metallicity stars which make up the halo are probably the simplest of all stars to model. However, there are several puzzling discrepancies between theory and observations. In this review, I will discuss a few problems which point to the need for improved stellar evolution models of halo stars. Observations of the surface abundance of  $^7\text{Li}$  provide a sensitive diagnostic of the internal structure of cool stars. Current stellar evolution models do not match the observed  $^7\text{Li}$  abundance patterns, suggesting that the input physics and/or the assumptions used in constructing the models are in need of revision. It appears that all halo stars have suffered some  $^7\text{Li}$  depletion, implying that the primordial  $^7\text{Li}$  abundance is higher than that presently observed in hot halo stars. Observations of abundances of various elements in globular cluster giant branch stars have suggested for some time now that some form of deep mixing, which is not present in theoretical models, occurs in halo stars. The driving mechanism for this mixing, and its incorporation into stellar models remain one of the key problems in stellar modeling. Current theoretical isochrones are able to provide a good match to observed colour-magnitude diagrams. However, there is some evidence that the theoretical luminosity functions are in disagreement with observations. This is an area which requires further study, as it suggests that the relative main sequence/giant branch lifetimes predicted by the models are incorrect. A discussion of some of the uncertainties involved in determining the ages of globular clusters is presented. The absolute ages of globular clusters provide a lower bound to the age of the universe, and so are of great interest to cosmologists. Unfortunately, present uncertainties in stellar models lead to a rather large range in the inferred ages of globular clusters of 11 – 18 Gyr.

## 1 Introduction

Halo stars are metal-poor ( $[\text{Fe}/\text{H}] \lesssim -1.0$ ) stars which typically have high velocity, randomly oriented orbits. As such, they were among the first objects formed in the Milky Way and so provide important constraints on the early formation history of the Milky Way, and galaxy formation in general. In addition, the fact that halo stars were formed soon after the big bang implies that determining a reliable estimate for their age yields a reasonable minimum estimate for the age of the universe. Finally, the primordial abundance of  $^7\text{Li}$  may be estimated by observations of halo stars coupled with stellar evolution calculations. As  $^7\text{Li}$  is produced via big bang nucleosynthesis, an accurate determination of its primordial abundance provides a diagnostic of the conditions in the very early universe.

The importance of halo stars to astrophysics has long been recognized, and theoretical stellar evolution models of halo stars have been calculated for many years. A major uncertainty in these early stellar models, which remains today, is the treatments of convection. A detailed understanding of the structure in the transition region between radiative and convective regions continues to be one of the key quests in stellar astrophysics. Fortunately, for the main sequence and red giant branch stars which are in the halo, convection is only important in the outer layers. In this review, I will concentrate on these early stages of stellar evolution, emphasizing the problems which are encountered with current stellar evolution models of halo stars when they are compared to observations. The more advanced stages of stellar evolution are covered by Lattanzio and Dorman in this volume. Due to space limitations, I have chosen to focus on four areas of halo star stellar evolution research. Section 2 discusses the importance of  ${}^7\text{Li}$  for constraining stellar evolution models, and big bang nucleosynthesis. The long standing difficulty in understanding the abundance anomalies observed in red giant branch stars is highlighted in Sect. 3. Section 4 will assess the current status of theoretical isochrone and luminosity function calculations. A discussion of the uncertainties in globular cluster ages estimates is presented in Sect. 5. A brief summary is presented in Sect. 6.

## 2 ${}^7\text{Li}$

Observations of lithium<sup>1</sup> provide a critical test of stellar models, for  ${}^7\text{Li}$  is destroyed at temperatures around  $2.6 \times 10^6$  K, which is located in the outer envelope of most halo stars. Hence, its observed surface abundance can be dramatically affected by small changes in the convection zone depth, or small amounts of mixing in the stellar radiative regions below the surface convection zone. In addition to its importance for testing stellar models,  ${}^7\text{Li}$  is interesting because it was produced in significant quantities during the big bang. Thus, an accurate determination of the primordial abundance of  ${}^7\text{Li}$  serves as a test of big bang nucleosynthesis (eg. Smith, Kawano & Malaney 1993). For these reasons, there have been a large number of theoretical and observational studies of  ${}^7\text{Li}$  in halo stars.

Observational studies have revealed a remarkably uniform  ${}^7\text{Li}$  abundance of  $\log N(\text{Li}) \simeq 2.2$  (where  $\log N(\text{Li}) \equiv \log(\text{Li}/\text{H}) + 12$ ) over a wide range of effective temperature and metallicity ( $[\text{Fe}/\text{H}] \lesssim -1.5$ ,  $T_{\text{eff}} \gtrsim 5700$  K; Spite & Spite 1982; Thorburn 1994). This  ${}^7\text{Li}$  plateau provides a severe constraint for any mixing mechanism which operates in the radiative regions of a star, for such a mixing mechanism must provide a nearly uniform depletion of  ${}^7\text{Li}$  over a wide range of effective temperatures and metallicities. This constraint is often assumed to be so strong as to preclude any  ${}^7\text{Li}$  depletion in halo stars, and so the observed abundance is assumed to represent the primordial abundance (eg. Walker *et al.* 1991). However, as shown by Boesgaard (1991)  *${}^7\text{Li}$  has been depleted in all Pop I stars which possess a surface convection zone.* As the structure of the plateau stars is similar to the Pop I stars, this is a strong argument in favour of  ${}^7\text{Li}$  depletion occurring in the plateau stars. In addition, we note that the study of

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<sup>1</sup>In stars, Li may exist in two different isotopes:  ${}^6\text{Li}$  and  ${}^7\text{Li}$ . It is extremely difficult observationally to distinguish between  ${}^6\text{Li}$  and  ${}^7\text{Li}$ , thus the observations typically determine the total Li present in a star. However,  ${}^6\text{Li}$  is even more fragile than  ${}^7\text{Li}$  and so is likely to be destroyed in most stars. In addition, Smith, Lambert & Nissen (1992) determined the abundance ratio of  ${}^6\text{Li}/{}^7\text{Li}$  to be  $0.05 \pm 0.02$  in the halo star HD 84937. It appears that the total Li content in a star is dominated by  ${}^7\text{Li}$ . Thus, we will assume that the Li observations actually measure  ${}^7\text{Li}$ .

Figure 1: Comparison of Kurucz and grey atmosphere  ${}^7\text{Li}$  isochrones to the data from Thorburn (1994). The isochrones shown have an age of 18 Gyr and an overshoot of 0.02 pressure scale heights. The initial  ${}^7\text{Li}$  abundance was taken to be 2.25.

Thorburn (1994) has revealed that about 5% of the halo stars with effective temperatures in the plateau regions have  ${}^7\text{Li}$  abundances which are depleted by at least an order of magnitude. This clearly indicates that large  ${}^7\text{Li}$  depletions occur in some ‘plateau’ stars, and presents a further challenge to theoretical models.

For halo stars which are cooler than the plateau ( $T_{\text{eff}} \lesssim 5600\text{ K}$ ), significant amounts of  ${}^7\text{Li}$  depletion has occurred. This provides a further test for stellar models, for the amount of  ${}^7\text{Li}$  depletion in cool star halo models is very sensitive to the depth of the surface convection zone. Theoretical models predict that this  ${}^7\text{Li}$  depletion occurs largely on the pre-main sequence, where the convection zone has its greatest depth. Figure 1 compares  ${}^7\text{Li}$  depletions predicted by standard stellar models with the latest available input physics (opacities, nuclear generation rates, etc.) from Chaboyer & Demarque (1994) to the observations of Thorburn (1994). Two different theoretical models are shown; one set uses a grey model atmosphere for the surface boundary condition while the other set uses the Kurucz (1992) model atmospheres. These models match the bulk of the plateau star observations, though they are unable to account for the depleted plateau stars. However, the models which use the Kurucz (1992) atmospheres predict very little  ${}^7\text{Li}$  depletion in cool stars, in clear contrast to the observations. Thus, stellar models with the best available input physics are unable to match the observed  ${}^7\text{Li}$  abundances in halo stars. This is also true for models which incorporate extra mixing in the radiative regions (Chaboyer & Demarque 1994). Note that models which employ the grey model atmospheres do a better job of matching the observations, but still predict too little  ${}^7\text{Li}$  depletion in the cool stars. It is clear that improvements in the basic physics (model atmospheres, opacities, etc.) used in stellar models of halo stars are required.

Figure 2: Comparison of theoretical models to main sequence (Thorburn 1994) and sub-giant observations (Pilachowski *et al.* 1993).

Once a star evolves off the main sequence, the convection zone deepens and material which had previously destroyed its  ${}^7\text{Li}$  becomes part of the convection zone. This results in a dilution of the surface  ${}^7\text{Li}$  abundance for stars on the sub-giant and red giant branch. Thus, observations of  ${}^7\text{Li}$  in halo sub-giants can be used to test if the convection zone depth in the models agrees with reality. Pilachowski, Sneden & Booth (1993) obtained an extensive data set which is plotted in Fig. 2 along with the main sequence observations of Thorburn (1994), and the theoretical predications. As was clear from Fig. 1, the main sequence models predict too little depletion at cool temperatures. In contrast, the sub-giant models *over-deplete*  ${}^7\text{Li}$  in the range 5000–5700 K. This suggests that the convection zone depth is incorrect in these sub-giant models, though in an opposite sense to the problem which occurs for the main sequence models. This points to the need for improved physics in halo star stellar models. In addition, we note that the coolest sub-giants ( $T_{\text{eff}} < 4800$  K) have depleted considerably more  ${}^7\text{Li}$  than is predicted by the models. This suggests that some form of extra mixing occurs on the giant branch, a point which is discussed in more detail in Sec. 3.

The near uniformity of the plateau star  ${}^7\text{Li}$  abundances, which is predicted by the standard models, puts severe constraints on mixing in stellar radiative zones. For example, microscopic diffusion predicts greater  ${}^7\text{Li}$  depletions at hotter effective temperatures. This is clearly ruled out by the observations (Chaboyer & Demarque 1994), and so either diffusion does not operate in halo stars, or some other mechanism acts to counteract the effects of diffusion on the  ${}^7\text{Li}$  abundances. Given the increasingly strong evidence from helioseismology that diffusion is occurring in the Sun (eg. Christensen-Dalsgaard, Proffitt & Thompson 1993), this latter approach has been investigated by two groups. Vauclair & Charbonnel (1995) found that stellar winds of order  $10^{-12.5} \text{ M}_{\odot} \text{ yr}^{-1}$  coupled with microscopic diffusion were able to nat-

urally explain the plateau star  ${}^7\text{Li}$  abundances, and implied a primordial  ${}^7\text{Li}$  abundance of  $\log N(\text{Li}) = 2.5 \pm 0.1$ . These models also predict a small dispersion in  ${}^7\text{Li}$  abundance among the plateau stars, and suggest that the severely depleted plateau stars had significantly higher mass loss rates. Chaboyer & Demarque (1994) found that stellar models which coupled mixing induced by rotational instabilities with microscopic diffusion provided a reasonable match to the plateau star  ${}^7\text{Li}$  abundances, and predicted a primordial  ${}^7\text{Li}$  abundance of  $\log N(\text{Li}) \simeq 3.1$ . These models also predict that a real dispersion in  ${}^7\text{Li}$  abundance should exist among the plateau stars.

Standard models predict that no dispersion exists in the  ${}^7\text{Li}$  abundances of the plateau stars. Thus, determining whether or not a dispersion exists is a key test which differentiates standard models from models which include mixing in stellar radiative regions and hence, predict a primordial  ${}^7\text{Li}$  abundance which is higher than that observed in halo stars. It is clear from present observations that if a dispersion in  ${}^7\text{Li}$  abundances exists, it must be rather small. Analysis of the dispersion is complicated by the fact that the conversion from measured equivalent widths to actual  ${}^7\text{Li}$  abundances is a strong function of the effective temperature of the star. In addition, one has to accurately determine the error in each individual observation.

Deliyannis, Pinsonneault & Duncan (1993) presented an analysis of the dispersion in the colour-equivalent width plane and found that a real dispersion exists, at the  $\sim 20\%$  level. Much of the uncertainty surrounding the determinations of the dispersion can be overcome by observing stars (with the same instrumental setup) which have identical colours, reddenings (hence effective temperature), and ages. Deliyannis, Boesgaard & King (1995) have recently performed such a test, by observing 3 sub-giants in the globular cluster M92 with the Keck telescope. By observing stars in a globular cluster, they were able to measure  ${}^7\text{Li}$  abundances in stars which have identical colours, reddenings and ages. Hence, any difference in the observed equivalent width of  ${}^7\text{Li}$  is direct evidence of a dispersion in the  ${}^7\text{Li}$  abundances. They found that a dispersion in  ${}^7\text{Li}$  abundance does indeed exist among the plateau stars, which is evidence that  ${}^7\text{Li}$  depletion has occurred among the plateau stars. Note however, that Deliyannis *et al.* (1995) observed sub-giant stars with  $T_{\text{eff}} \sim 5700$  K, which is near the cool edge of the plateau. It would be best to obtain similar observations for hotter stars, to ensure that these sub-giants have not suffered dilution of  ${}^7\text{Li}$  at the surface due to the deepening of the surface convection zone.

### 3 Mixing on the Red Giant Branch

For a number of years, there has been evidence that CNO processed material (which is enhanced in nitrogen, and depleted in carbon) is being transported to the surface of red giant stars in the halo. Perhaps the best evidence for this comes from the observations of carbon in M92 giants by Langer *et al.* (1986). They found that the carbon to iron ratio ( $[\text{C}/\text{Fe}]$ ) smoothly decreased from  $[\text{C}/\text{Fe}] = +0.1$  around the main sequence turn-off to about  $[\text{C}/\text{Fe}] = -1.0$  for stars four magnitudes brighter than the turn-off, as shown in Fig. 3. However, standard stellar evolution models predict that the base of the convection zone is never deep enough to dredge up CNO processed material, and hence,  $[\text{C}/\text{Fe}]$  should remain constant. The fact that the decrease of  $[\text{C}/\text{Fe}]$  is smoothly related to the absolute magnitude of the stars on the giant branch clearly points to the fact that some non-standard, slow mixing is occurring in the radiative regions of giant stars. Further observational evidence has been found by Dickens *et al.* (1991) who

Figure 3: Carbon abundances in M92 sub-giants and red giants as a function of absolute magnitude, from Langer *et al.* (1986). Only high quality observations are plotted. The dotted line is a simple least squares fit to the data:  $[C/Fe] = -0.67 + 0.23 M_V$ .

determined the abundances of carbon, nitrogen and oxygen in giants in the globular cluster NGC 362. They found that  $[C/Fe]$  ranged from 0 to  $-1$ . However, the total amount of C+N+O (which is preserved in the CNO cycle) was constant. Hence, the range in the observed carbon abundances can only be explained by the fact that CNO processed material has been brought to the surface in these giant stars.

In addition to the above observations, I should note that there are some observations of abundance patterns which are best explained by primordial variations. A recent review by Kraft (1994) provides a detailed description of all of the abundance anomalies found by observers in metal-poor giant stars. For example, there is a global anti-correlation between oxygen and sodium abundances in globular cluster red giants (Kraft *et al.* 1993). The most obvious explanation for such a anti-correlation is due to primordial variations in oxygen and sodium. However, Denissenkov & Denisenkova (1990) have suggested that sodium could be produced via proton capture on neon in the CNO burning region of a red giant star. Thus, the global oxygen-sodium anti-correlation could be further evidence that CNO processed material is being transported to the surface of red giant stars. This possibility has been studied by Langer *et al.* (1993) and Denissenkov & Weiss (1995) who utilized an *ad hoc* slow mixing prescription coupled with a detailed nuclear reaction network to predict changes in the surface abundances as a star evolved up the red giant branch. Both groups concluded that it is possible to reproduce the global oxygen-sodium anti-correlation by assuming that CNO processed material is being brought to the surface of red giant stars.

Given the large number of observations which clearly indicate that CNO processed material is dredged up to the surface of giant stars, in contradiction with standard stellar models, it is

clear that the theoretical models need to be improved. These models must incorporate some form of slow mixing in the radiative regions, which will progressively bring more CNO processed material to the surface of the star as it evolves up the giant branch. To do this, the driving mechanism for the mixing must first be identified. Sweigart & Mengel (1979) proposed that meridional circulation (induced by rotation) was responsible for the mixing between the base of the surface convection zone, and the CNO processed material in giant branch stars. The models of Sweigart & Mengel (1979) were able to reproduce (and indeed, predict) most of the observations provided that (i) the convective envelope does not rotate as a solid body and (ii) substantial reservoirs of angular momentum exist buried inside main-sequence stars. This first assumption appears to be reasonable, given that the convective turnover times are long compared to the size of the convection zone and the rotation velocities. However, the second assumption clearly needs to be explored in more detail. Indeed, there has been little theoretical work on mixing in the radiative regions of giant branch stars in recent years. In view of the wealth of observational evidence which is now available, this is an area which clearly requires renewed attention from stellar evolution theorists.

## 4 Isochrones and Luminosity Functions

The calculation of isochrones and luminosity functions play a vital role in many areas of astrophysics. As such, they probably represent the most widely-used output of stellar evolution calculations. Luminosity functions and isochrones are used in a variety of applications: estimating the ages of star clusters and individual stars; as basic building blocks for stellar population modeling and galactic evolution codes; and even to determine the Hubble constant, via the surface brightness fluctuation technique. Modern isochrones provide a good fit to colour-magnitude diagrams in globular clusters. A typical example is shown in Fig. 4, which compares the recent set of Yale isochrones (Chaboyer *et al.* 1995) to observations of M68 (Walker 1994) in both  $B - V$  and  $V - I$ . A good fit is obtained to the 16 Gyr isochrone in both colours, without any *ad hoc* colour shifts. These isochrones use a solar calibrated mixing length ( $\alpha = 1.7$ ); if the mixing is changed by as little as 0.2 then it is no longer possible to obtain a good fit. While such good fits are important to those who use isochrones for stellar population modeling, the freedom to choose the mixing length imply that fitting theoretical isochrones to observed colour-magnitude diagrams is not a good test of the stellar evolution models.

A much more stringent test of stellar models is obtained by comparing theoretical luminosity functions to observed number counts of stars in globular clusters. If one focuses on the region from just below the main sequence turn-off (within about 1 mag) to the tip of the giant branch, the theoretical luminosity functions are independent of the initial mass function and are only a function of the relative stellar lifetimes in the various phases of evolution. The stellar lifetimes are a remarkably robust prediction of the theoretical models. A comparison between the Revised Yale isochrones (Green, Demarque & King 1987; based on stellar models calculated in the 1970's), and the most recent Yale set reveals virtually no differences. Unfortunately, there are very few good quality observations of luminosity functions. This is due to the fact that present CCDs have small physical sizes, and the construction of luminosity functions requires good photometry from below the main sequence turn-off to the tip of the giant branch, over a large area of a globular cluster. Although there were some hints of discrepancy between

Figure 4: Comparison of theoretical isochrones to observations of M68 by Walker (1994). A reasonable fit is obtained in both  $B - V$  (left panel) and  $V - I$  without any *ad hoc* colour shifts being applied to the isochrones.

the theoretical models and observations in the past, in general the agreement was satisfactory. However, Bolte (1994) has recently published a very high quality luminosity function of M30 ( $[\text{Fe}/\text{H}] = -2.1$ ) which is clearly in disagreement with theoretical calculations, as shown in Fig. 5. If the theoretical luminosity functions are normalized to the main sequence number counts, then there is about a 15% excess in the number of observed stars on the giant branch. It is important to note that this discrepancy is present for all reasonable choices of the assumed distance modulus, reddening or metallicity. In addition, as the luminosity function is a rather robust prediction of the theoretical models, it does not matter whose set of luminosity functions are used to compare to the observations – the discrepancy between theory and observations is always present. If the observations are interpreted at face value, this suggests that the relative main sequence/giant branch lifetimes are in error by about 15%. I caution however, then M30 is a cluster which has undergone considerably dynamical evolution, so it is important to have these observations verified in other globular clusters. Such observational efforts are now underway by Peter Stetson and the Yale group. If their results confirm Bolte’s (1994) observations, then a considerable revision in the relative main sequence/giant branch lifetimes in halo stars will be necessary. The study of luminosity functions is clearly an area worthy of further theoretical and observational effort.



Figure 5: Comparison of a theoretical luminosity function from Chaboyer *et al.* (1995) to observations in M30 by Bolte (1994). The theoretical luminosity function has been normalized to the main sequence ( $5 \leq M_V \leq 6$ ) number counts. Note the excess of observed stars ( $\sim 15\%$ ) on the red giant branch ( $M_V \lesssim 1$ ) as compared to the observations.

## 5 Globular Cluster Ages

The determination of accurate globular cluster ages are important in furthering our understanding of the formation of the Milky Way, and in providing a lower limit to the age of the universe. This latter application has received increasing attention lately, as the rather old ages inferred for globular clusters ( $\sim 16$  Gyr) appear to be in conflict with the young age inferred from measurements of the Hubble constant, and the density of the universe (eg. Freedman *et al.* 1994). If the stellar ages are correct, then a rather dramatic revision of the preferred cosmological model is in order (Leonard & Lake 1995). As such, it is important for the stellar evolution theorist to provide a reasonable estimate of the possible error in the determination of the absolute ages of globular clusters.

Globular cluster ages are determined by comparing theoretical isochrones to observed globular cluster colour magnitude diagrams. There are a variety of ways in which this can be accomplished. In assessing the usefulness of the different methods, it is important to recall that the colours predicted by the isochrones are subject to large uncertainties due to our poor understanding of how to treat convection in the superadiabatic regions near the surface, and due to our limited knowledge of stellar atmospheres. The  $\Delta(B - V)$  technique uses the difference in colour between the main sequence turn-off and the base of the giant branch as an age diagnostic (Sarajedini & Demarque 1990; VandenBerg, Bolte & Stetson 1990). This technique is a sensitive test which can determine if age differences exist between clusters of similar metallicity, but cannot be relied upon to determine absolute ages, age differences between clusters of different metallicity, or the absolute size of the age difference for clusters of the same metallicity.

The most accurate theoretical ages are determined via methods which rely on using the

TABLE 1: GLOBULAR CLUSTER  
AGES ERROR BUDGET

Description	Error ( $\pm\%$ )
$M_v(\text{RR})$	13
$[\alpha/\text{Fe}]$	7
$^4\text{He}$ abundance	3
Treatment of convection	9
Diffusion	4
Colour Transformation	3
Nuclear Reaction Rates	3

luminosity of the isochrones (and not colour) as the age indicator. A popular approach is to use the difference in magnitude between the horizontal branch and the main sequence turn-off ( $\Delta V_{\text{HB}}^{\text{TO}}$ ). In addition to being a luminosity based age determination, this method has the advantage of being independent of reddening. Unfortunately, the precise location of the main sequence turn-off, and indeed the magnitude of the horizontal branch in the instability strip can be difficult to determine observationally. For this reason, observational errors alone lead to typical  $1\sigma$  errors of  $\sim 10\%$  in the derived ages. For the purposes of providing a reliable estimate of the minimum age of the universe, this difficulty can be overcome by determining the mean age of a number of globular clusters, thereby minimizing the observational errors.

From the theoretical point of view, determination of the absolute magnitude of the horizontal branch is fraught with difficulties. Horizontal branch stars are convective in the energy producing regions, which leads to large uncertainties in the models. Fortunately, there are several observationally-based techniques which can be used to determine the absolute magnitude of RR Lyr stars ( $M_v(\text{RR})$ ), and hence, the horizontal branch. These calibrations can be combined with theoretical isochrones which determine the absolute magnitude of the turn-off to produce a semi-empirical calibration of  $\Delta V_{\text{HB}}^{\text{TO}}$  as a function of age and metallicity. The various observational calibrations of  $M_v(\text{RR})$  agree to within  $\sim 0.25$  mag (for a discussion of  $M_v(\text{RR})$  see Carney, Storm & Jones 1992; Chaboyer, Demarque & Sarajedini 1996). This directly translates into an overall error in the age determination of about 25%.

This large uncertainty in the derived ages has been widely discussed in the literature (eg. Renzini 1991). However, not much attention has been focused on the total error budget for the age determinations, including possible errors due to the theoretical models. I have recently undertaken such an investigation (Chaboyer 1995). The mean age of the oldest globular clusters was determined using a number of different isochrones, each of which were constructed using different assumptions for the input physics. I found that the opacities, model atmospheres, convective overshoot, and uncertainties in the globular cluster metallicity scale have little or no impact on the derived ages. A potentially large source of error involved the equation of state. Whether or not one included the Debye-Hückel correction into the equation of state changed the ages by 7%. Since then, I have incorporated the OPAL equation of state tables (Rogers 1994), and found that it gives gives nearly identical results to the equation of state

Figure 6: The density of the universe  $\Omega$  (in units of the critical density) is plotted as a function of the age of the universe, for various values of the Hubble constant (in units of km/s/Mpc), assuming a zero cosmological constant. Current estimates for the absolute ages of the globular clusters are in a range 11 – 18 Gyr, which provides a minimum estimate for the age of the universe.

with the Debye-Hückel correction (Chaboyer & Kim 1995). Given the good agreement between the sophisticated OPAL equation of state results, and the Debye-Hückel equation of state, I no longer believe that the equation of state leads to significant uncertainties in the derived ages. Using these new equation of state results, combined with the ages published in Chaboyer (1995), my best estimate for the mean age of the oldest globular clusters is 14.5 Gyr. The error budget associated with this number is tabulated in Table 1.

The percentage errors presented in Table 1, are *not* Gaussian 1- $\sigma$  error bars, but are meant to represent my best estimate of the total possible error associated with a given uncertainty. Next to the uncertainty associated with  $M_V(\text{RR})$  the largest single source of error is the treatment of convection. This is somewhat surprising, as it is often assumed that the  $\Delta V_{\text{HB}}^{\text{TO}}$  technique yields ages which are independent of the mixing length theory of convection used in stellar models. While it is true that the lifetime of the stars do not depend on the mixing length, I found that changes in the mixing length changed the shape of the isochrones around the turn-off, resulting in different magnitudes for the turn-off (defined to be the bluest point). It may be possible to eliminate this uncertainty, by defining the turn-off in terms of the luminosity function, as the number of stars drop dramatically around the turn-off. Another important source of error in the age determination is the amount by which the  $\alpha$ -capture elements (O, Mg, Si, S, and Ca) are enhanced over their solar ratio. Changing  $[\alpha/\text{Fe}]$  by 0.2 dex results in a 7% change in the derived age. Given the possible errors listed in Table 1, and recalling that my best estimate for the oldest globular clusters are 14.5 Gyr old, it is reasonable to conclude that the true age of the oldest globular clusters lies in the range 11 – 18 Gyr. The implications of this age estimate for cosmology are shown in Fig. 6, where the density of the universe  $\Omega$ , is plotted as a function

of age, for various values of the Hubble constant (assuming no cosmological constant). As there is general consensus that observations require  $\Omega \gtrsim 0.3$ , the minimum age of 11 Gyr requires that a Hubble constant less than  $\sim 70$  km/s/Mpc. The best estimate for the minimum age of the universe (14.5 Gyr) requires that  $H_0 \lesssim 55$  km/s/Mpc. Although there is still considerable debate concerning a true value of the Hubble constant, recent evidence appears to be favouring the higher values (eg. Freedman *et al.* 1994; Riess, Press & Kirshner 1995). This suggests that a non-zero cosmological constant is required (Leonard & Lake 1995).

The rather large error in the globular cluster ages significantly reduces the constraints imposed by the ages on cosmology. Reducing the size of this error will require an improved understanding of the possible sources of error listed in Table 1. Given the relative sizes of the errors, effort should be concentrated on obtaining improved estimates of  $M_v(RR)$ ,  $[\alpha/Fe]$ , and a better understanding of how to treat convection in stars. Better knowledge of diffusion, nuclear reaction rates, and model atmospheres (used to derive the colour transformation) are needed if one wishes to reduce the error in the globular cluster age estimates below 10%.

## 6 Summary

This review of the evolution of halo stars has highlighted some inconsistencies between present stellar models and observations. These inconsistencies merit further study. As shown in Fig. 2, cool stellar models ( $T_{\text{eff}} \lesssim 5600$  K) which utilize the best available input physics deplete too much  $^7\text{Li}$  on the sub-giant branch, and too little  $^7\text{Li}$  on the main sequence. This implies that the convection zone depth is likely to be incorrect in these models. The plateau in  $^7\text{Li}$  abundances above  $T_{\text{eff}} \simeq 5700$  K provides a stringent test for any possible mixing which occurs in stellar radiative regions. Standard stellar models are unable to explain the  $\sim 5\%$  of stars in the plateau region which are severely depleted in  $^7\text{Li}$ . In addition, there is evidence for a dispersion in  $^7\text{Li}$  abundances for stars with identical effective temperatures and ages (Deliyannis *et al.* 1995). Such a dispersion is not predicted by standard models, and is evidence that additional mixing, and hence,  $^7\text{Li}$  depletion occurs in all halo stars. Further evidence that  $^7\text{Li}$  depletion has occurred in plateau stars is found via a comparison to Pop I stars – all of which show definite evidence for depleting  $^7\text{Li}$  (Boesgaard 1991). Stellar models which include microscopic diffusion and a stellar wind (Vauclair & Charbonnel 1995), or diffusion and rotational mixing (Chaboyer & Demarque 1994) do a good job of reproducing the observed plateau star  $^7\text{Li}$  abundances and suggest that the primordial  $^7\text{Li}$  abundance at least 0.3 dex higher than what is presently observed in the plateau stars.

Observations of abundance anomalies in globular cluster red giant stars have demonstrated for some time that some form of slow mixing operates in the radiative regions of stars. This evidence is plotted in Fig. 3 which demonstrates that CNO processed material (which is depleted in carbon) is slowly being brought to the surface of red giant stars. This is not predicted by standard stellar models. Even though there has been a steady accumulation of observational evidence demonstrating the failure of standard stellar models over the last several years, there has been little theoretical work to improve the stellar models. Sweigart & Mengel (1979) proposed that meridional circulation (induced by rotation) was responsible for mixing CNO processed material into the convection zone (and hence, to the surface) in giant branch stars. Their models were in reasonable agreement with the observations, and should be pursued further

in light of the extensive observational database which is now available.

Theoretical isochrones which are calculated from modern stellar evolution models provide a good match to observed colour-magnitude diagrams (cf. Fig. 4). However, there is some evidence that the luminosity functions do not match the observations (Fig. 5), predicting relative main sequence/giant branch lifetimes which are in error by  $\sim 15\%$ . This merits further observational and theoretical study, for this may have important implications for stellar evolution models. Determining the age of the oldest globular clusters provides the best estimate for the minimum age of the universe, and is of great interest to cosmologists. Currently, the best estimate yields an age of 14.5 Gyr for the oldest globular clusters. A critical evaluation of the possible error associated with this estimate (Table 1, see Chaboyer 1995) suggests that the true age of the oldest globular clusters lies in the range 11 – 18 Gyr. This rather large allowed range in the globular cluster ages significantly reduces the constraints imposed on cosmology. Reducing the error in the ages to below the 10% level will require an improved understanding of all of the possible error sources listed in Table 1.

The study of the evolution of halo stars has an impact on many areas of astrophysics, helping us to understand questions related to big bang nucleosynthesis, cosmology, and galaxy formation. The examples highlighted in this review illustrate the fact that our knowledge of the evolution of halo stars is incomplete. Given the broad ranging applications of halo star models, an improved knowledge of the evolution of halo stars is highly desirable and worthy of further study.

## Acknowledgements

I would like to thank H. Craig for her careful editing of this manuscript.

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## Discussion

**I. Roxburgh:** If the inner 30% or so of the mass of low mass stars was mixed on the main sequence, would one predict results that were in conflict with observations (or more in conflict than standard models)?

**B. Chaboyer:** Mixing the inner part of a halo star is essentially assuming that such stars have convective cores. Stars which have convective cores display a characteristic hook in colour-magnitude diagram as they leave the main sequence. These hooks are observed in colour-magnitude diagrams of young star clusters, as predicted by stellar models. However, globular cluster colour-magnitude diagrams contain no such hooks, which is a strong argument against substantial mixing occurring in the inner region of low mass stars.

**B. Dorman:** I would like to comment that the zero-point of the  $M_V(RR)$  relation is not as clear-cut as you seem to paint it. The evolutionary models with  $Y = 0.23$  (which give the bright

zero-point) are consistent with primordial helium, and R-method determination of the helium abundance in globular clusters. The brighter Walker (1992) zero-point is apparently supported also by recent HST observations of M31 clusters (Faber 1995, private communication). It is supported by Carney *et al.* (1992) re-analysis of the main-sequence fitting technique. Further, the Baade-Wesselink analysis may be subject to a systematic underestimate in the radius (eg. Castellani & Santis 1994). Statistical parallax methods that are independent of the Baade-Wesselink distances give results (Barnes & Hawley 1986) that are consistent within the errors with the brighter distance scale.

**B. Chaboyer:** I agree with you that the question of the zero-point in the  $M_V(\text{RR})$  relation is still a matter of debate. As such, I have assumed an uncertainty in the zero-point of 0.25 mag, which is the typical difference between the ‘bright’ and ‘faint’ calibrations of the  $M_V(\text{RR})$  zero-point. I am not aware of the HST M31 observations, but I should point out that Walker’s (1992) value of the zero-point was based on an LMC distance modulus of 18.5 mag. Gould (1995) has re-analyzed the distance to the LMC based on the rings surrounding SN1987A, and determined a lower limit to the distance modulus which is 18.37 mag. This makes Walker’s zero-point *fainter* by 0.13 mag, removing much of the discrepancy with the statistical parallax results. My understanding of the Carney *et al.* (1992) main-sequence fitting results is rather different than yours. I believe that they determined a zero-point which was even fainter than the statistical parallax work, and incompatible with Walker’s (1992) value. I agree that the Baade-Wesselink analysis may be subject to systematic uncertainties and is best used to determine the slope of the  $M_V(\text{RR})$  relation with metallicity, and not the zero-point.

**C. Charbonnel:** In our models (Vauclair & Charbonnel 1995) which include microscopic diffusion and stellar wind, we not only reproduce the slope of the lithium ‘plateau’ for halo stars, but, with a reasonable range of mass loss rates, we also reproduce the observed dispersion and explain the upper limits seen well below the plateau. So my question is: What is the primordial abundance that you predict with rotation-induced mixing, and how do you explain all of the characteristics of the plateau (slope, dispersion and upper limits) without *ad hoc* assumptions?

**B. Chaboyer:** Our models with rotation-induced mixing and microscopic diffusion predict a primordial abundance of  ${}^7\text{Li}$  which is 1 dex above the presently observed abundance. However, given the uncertainties in the models, I believe that a more accurate statement is that the  ${}^7\text{Li}$  in plateau stars has been depleted by at least 0.4 dex, implying a primordial abundance of  $\log N(\text{Li}) > 2.6$ . These models with rotation-induced mixing and microscopic diffusion are able to naturally explain the slope of the plateau, using the same parameters which are able to match  ${}^7\text{Li}$  observations in the open clusters and the Sun. Our models account for the dispersion in the plateau if we assume the majority of stars are born as slow rotators ( $\sim 10$  km/s), with about  $\sim 20\%$  having rotation velocities 30 km/s or greater. This implied distribution in initial rotation velocities is similar to what is observed in present day T-Tauri stars. The very low upper limits observed in about 5% of the halo star sample are more difficult to reconcile with our models. Perhaps they are a sign that a few stars were born with very high rotation velocities, or perhaps they are due to high mass loss rates as you suggest.

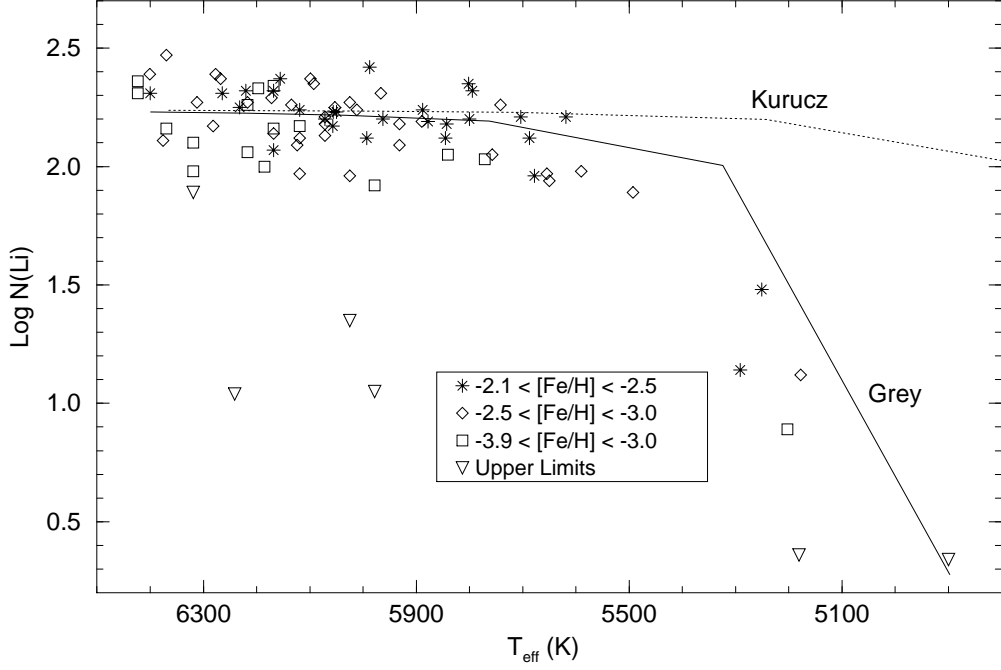


Figure 1: Comparison of Kurucz and grey atmosphere  ${}^7\text{Li}$  isochrones to the data from Thorburn (1994). The isochrones shown have an age of 18 Gyr and an overshoot of 0.02 pressure scale heights. The initial  ${}^7\text{Li}$  abundance was taken to be 2.25.

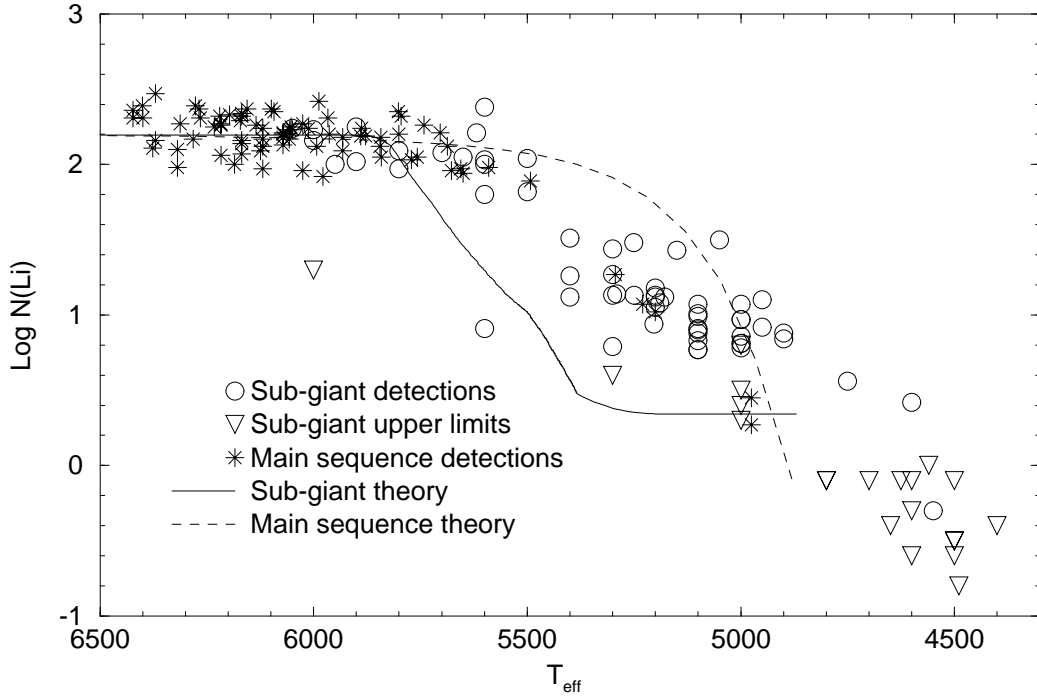


Figure 2: Comparison of theoretical models to main sequence (Thorburn 1994) and sub-giant observations (Pilachowski *et al.* 1993).



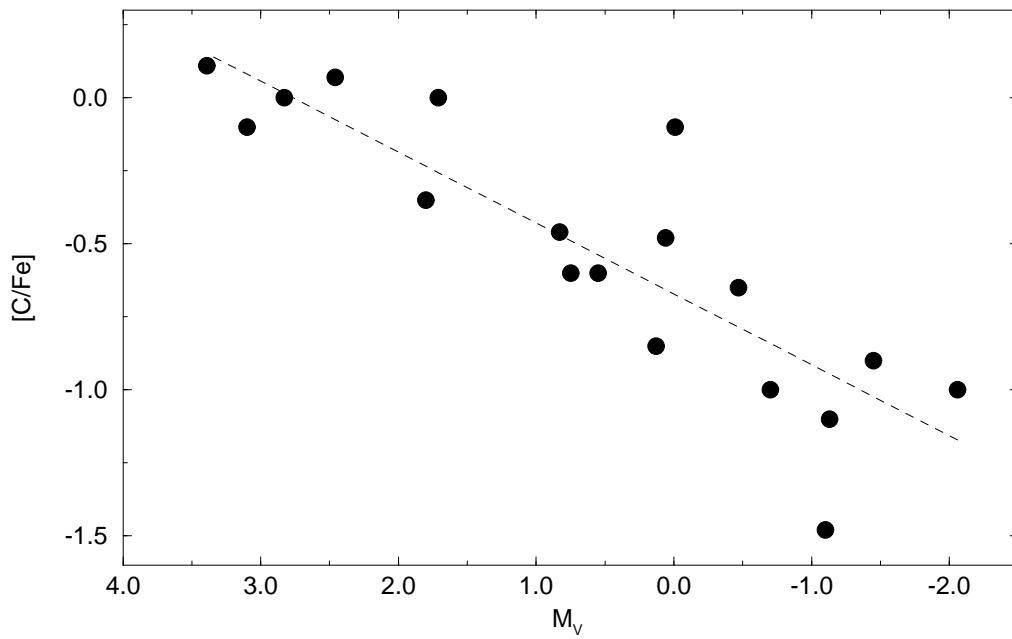


Figure 3: Carbon abundances in M92 sub-giants and red giants as a function of absolute magnitude, from Langer *et al.* (1986). Only high quality observations are plotted. The dotted line is a simple least squares fit to the data:  $[C/Fe] = -0.67 + 0.23 M_V$ .

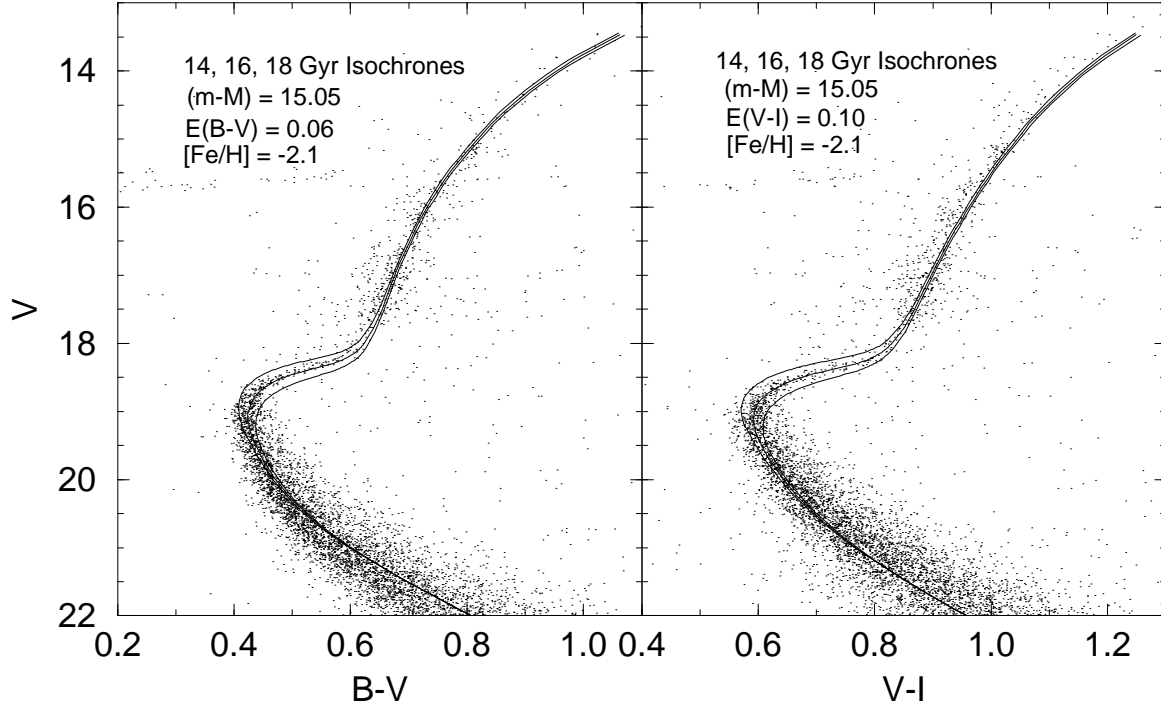


Figure 4: Comparison of theoretical isochrones to observations of M68 by Walker (1994). A reasonable fit is obtained in both  $B - V$  (left panel) and  $V - I$  without any *ad hoc* colour shifts being applied to the isochrones.

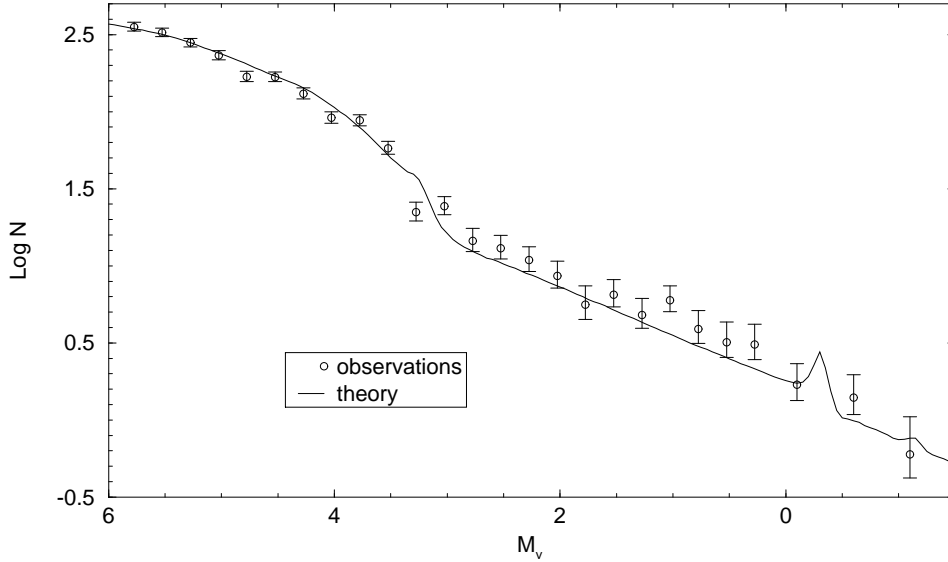


Figure 5: Comparison of a theoretical luminosity function from Chaboyer *et al.* (1995) to observations in M30 by Bolte (1994). The theoretical luminosity function has been normalized to the main sequence ( $5 \leq M_V \leq 6$ ) number counts. Note the excess of observed stars ( $\sim 15\%$ ) on the red giant branch ( $M_V \lesssim 1$ ) as compared to the observations.

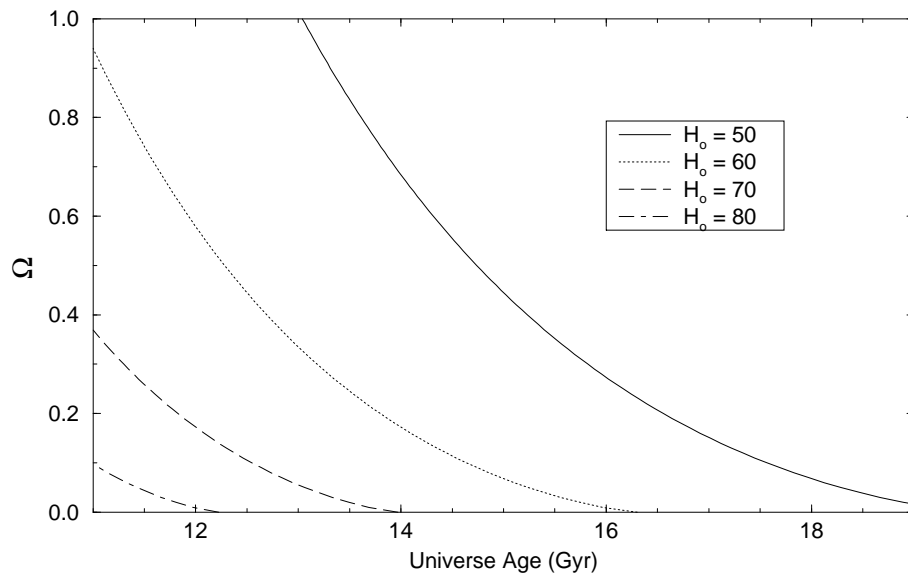


Figure 6: The density of the universe  $\Omega$  (in units of the critical density) is plotted as a function of the age of the universe, for various values of the Hubble constant (in units of km/s/Mpc), assuming a zero cosmological constant. Current estimates for the absolute ages of the globular clusters are in a range 11 – 18 Gyr, which provides a minimum estimate for the age of the universe.